

ALGORITHM 20  
REAL EXPONENTIAL INTEGRAL .

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**real procedure  
comment**

Expint (x) ; **real** x ;  
 $-E_i(-x) = \int_x^\infty (e^{-u}/u) du$  is computed for  
 $x > 0$  by approximation formulas. For  
 $0 < x < 1$  the approximation is from E. E.  
 Allen, Note 169, *MTAC* 56, pg 240 (1954).  
 The second approximation formula is for  
 $1 \leq x < \infty$  and is from C. Hastings, Jr.,  
 "Approximations For Digital Computers"  
 (Princeton University Press, Princeton,  
 New Jersey, 1955). The absolute error  
 $\epsilon(x)$  is  $|\epsilon(x)| < 2 \times 10^{-7}$  for  $0 < x < 1$   
 and  $|\epsilon(x)| < 2 \times 10^{-8}$  for  $1 \leq x < \infty$  ;

**begin**

**real** y, w, z ;  
**if** x < 1 **then**  
 z := (((.00107857  $\times$  x - .00976004)  $\times$  x  
 + .005519968)  $\times$  x - .24991055)  $\times$  x  
 + .99999193)  $\times$  x - .57721566 - ln(x)  
**else begin**  
 y := ((x + 8.5733287401)  $\times$  x  
 + 18.059016973)  $\times$  x + 8.6347608925)  $\times$  x  
 + .2677737343 ;  
 w := (((x + 9.5733223454)  $\times$  x  
 + 25.6329561486)  $\times$  x  
 + 21.0996530827)  $\times$  x + 3.9584969228 ;  
 z := exp (-x) /x  $\times$  (y/w) **end**  
 Expint := z **end**

(or to 8S for the 24.2 program) were rounded to 7S (or 8S).

After changing the constant .005519968 to .05519968, both programs gave acceptable accuracy over the range tested.

The 8S (24.2) program was compared with the 9D values given for  $-E_i(-x)$  in Mathematical Tables Project, *Tables of Sine, Cosine, and Exponential Integrals, Volume II* (1940) for the set of values  $x = 0.1(0.1)1.0(1.0)10.0$ . The largest discrepancy found was  $-16 \times 10^{-8}$  for  $x = 0.1$ . For  $x$  greater than 1, all values tested were good to 8S.

For computing real values of the exponential integral, this algorithm is much faster than EKZ (Algorithm 13). For  $x < 1$ , the ratio of speeds was of the order of 20.

\* Work supported by the U.S. Atomic Energy Commission.

REMARK ON ALGORITHM 20  
 REAL EXPONENTIAL INTEGRAL (S. Peavy, *Comm.*  
*ACM*, October 1960)

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A printing error has been called to our attention by J. A. Beutler of E. I. duPont de Nemours and Co. Lines 15 through 17 of Algorithm 20 should read

z := (((.00107857  $\times$  x - .00976004)  $\times$  x  
 + .05519968)  $\times$  x - .24991055)  $\times$  x  
 + .99999193)  $\times$  x - .57721566 - ln (x)

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CERTIFICATION OF ALGORITHM 20  
 REAL EXPONENTIAL INTEGRAL (S. Peavy, *Comm.*  
*ACM*, Oct. 1960)

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Expint (x) was programmed for the LGP-30 computer, using both a 7S floating-point compiler (ACT III) and an 8S floating-point interpretive code (24.2). Constants given to more than 7S