ALGORITHM 37
TELESCOPE 1

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```
procedure Telescope 1 (N, L, eps, limit, c)  ;  value limit, L  ;
          integer N  ;  real L, eps, limit  ;  array c  ;
comment:  Telescope 1 takes an Nth degree polynomial approxi-
```

mation $\sum_{k=0}^{N} c_k x^k$ to a function which was valid to within eps ≥ 0 over an interval (0, L) and reduces it, if possible, to a polynomial of lower degree, valid to within limit > 0. The initial coefficients c_k are replaced by the final coefficients, and the deleted coefficients are replaced by zero. The initial eps is replaced by the final bound on the error. N is replaced by the degree of the reduced polynomial. N and eps must be variables.

This procedure computes the coefficients given in the Techniques Department of the ACM Communications, Vol. 1, No. 9, from the recursion formula

$$a_{k-1} = -a_k \cdot \frac{k \cdot L \cdot (2k - 1)}{2(N + k - 1) \cdot (N - k + 1)} \quad ;$$

```
        begin integer k  ; array d[0:N]  ;
start:  if N < 1 then go to exit  ;  d[N] := -c[N]  ;
        for k := N step - 1 until 1 do
        d[k - 1] := -d[k] × L × k × (k - 0.5)/
          ((N + k - 1) × (N - k + 1))  ;
        if eps + abs (d[0]) < limit then
        begin eps := eps + abs (d[0])  ;
        for k := N step - 1 until 0 do c[k] := c[k] + d[k] ;
        N := N - 1  ; go to start end  ;
exit:   end
```

CERTIFICATION OF ALGORITHM 37
TELESCOPE 1 [K. A. Brons, *Comm. ACM*, Mar., 1961]

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The body of *Telescope 1* was compiled and tested on the LGP-30 using the Algol 60 translator system developed by the Dartmouth College Computer Center. No syntactical errors were found, and the program ran satisfactorily. The 10th degree polynomial obtained by truncating the exponential series was telescoped using lim! = $.1_{10} - 2$ and $L = 1.0$. The result was $N = 3$, *eps* = $.2103005_{10} - 3$, and coefficients +.9997892, −.9930727, +.4636493, − .1026781. The error curve for the telescoped polynomial was computed for $x = 0(.02)1.0$. The error extrema were bounded by *eps* to within 0.5%. The discrepancy is within the range of input conversion and round-off error.

CERTIFICATION OF ALGORITHM 37
TELESCOPE 1 [K. A. Brons, *Comm. ACM*, Mar. 1961]

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This procedure was tested on the CDC 160A, using 160A Fortran. The 10th degree polynomial obtained by truncating the series exp $(-x)$ was telescoped using $L = 1$ and lim = 0.001. The result was $N = 3$, eps = $0.21061862_{10} - 3$ and coefficients +0.99978965, −0.99307236, +0.46364955, −0.10267767. The error curve was computed for $x = 0(0.02)1.0$ and no error exceeded eps, the worst error being 2% of eps less than eps.

This result is in close agreement with that of Henry C. Thatcher, Jr. in his Certification (*Comm. ACM*, Aug. 1962). Mr. Thatcher has pointed out that he inadvertantly referred to the series for exp $(-x)$ as the "exponential series" thereby inferring the positive series exp $(+x)$. There is also a typographical error in his eps. It should be $+0.2103505_{10} - 3$.