ALGORITHM 38
TELESCOPE 2
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procedure  Telescope 2 (N, L, eps, limit, c)  ; value limit, L  ;
           integer N  ; real L, eps, limit  ; array c  ;
comment    Telescope 2 takes an Nth degree polynomial ap-
             proximation  sum_{k=0}^{N} c_k x^k  to a function which was
             valid to within eps ≥ 0 over an interval (−L, L)
             and reduces it, if possible, to a polynomial of
             lower degree, valid to within limit >0. The initial
             coefficients c_k are replaced by the final coefficients,
             and deleted coefficients are replaced by zero. The
             initial eps is replaced by the final bound on the
             error, and N is replaced by the degree of the re-
             duced polynomial. N and eps must be variables.
           This procedure computes the coefficients given in
             the Techniques Department of the ACM Com-
             munications, Vol. 1, No. 9, from the recursion
             formula

                  a_{k-2} = −a_k · k·L²(k − 1) / ((N + k − 2)·(N − k + 2))  ;

           begin integer k  ; real s  ; array d[0: N]  ;
start:     if N < 2 then go to exit  ; d[N] := −c[N]  ;
           for k := N step − 2 until 2 do
           d[k − 2]  := −d[k] × L↑2 × k × (k − 1)/
             ((N + k − 2) × (N − k + 2))  ;
           if (N/2) − entier (N/2) = 0 then s := d[0] else
             s := d[1]/N  ;
           if eps + abs(s) < limit then begin
             eps := eps + abs(s)  ;
           for k := N step − 2 until 0 do
             c[k] := c[k] + d[k]  ;
           N := N − 1  ; go to start end  ;
exit:      end
```

CERTIFICATION OF ALGORITHM 38
TELESCOPE 2 [K. A. Brons, *Comm. ACM*, Mar., 1961]
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This procedure was tested on the CDC 160A using 160A FORTRAN. The 10th degree polynomial obtained by truncating the series expansion of exp $(+x)$ was telescoped using $L = 1.0$ and lim = 0.001. The result was $N = 4$, eps = $0.59159949_{10} - 3$ and coefficients +1.0000447, +0.99730758, +0.49919675, +0.17734729, +0.043793910. Errors were calculated for $x = -1.0(0.02)1.0$. The only error to exceed eps was at $x = 1.0$ and was within 0.6% of eps.