

ALGORITHM 78
RATIONAL ROOTS OF POLYNOMIALS WITH IN-
TEGER COEFFICIENTS

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comment This ALGOL procedure, named *ratfact*, for finding rational roots of polynomials with integer coefficients is a pedagogical example illustrating the use of the **for** statement described in section 4.6.3. Also, an extension suggested by J. Peck of the well-known polynomial evaluation by nesting, i.e. Horner's method, is used. The polynomial $f(x) = a_0 + a_1x + \dots + a_nx^n$ with integer coefficients and with $a_0a_n \neq 0$ has a lowest term rational root p/q if and only if $a_0q^n + a_1q^{n-1}p + \dots + a_{n-1}q p^{n-1} + a_np^n = 0$, also q must be a factor of a_n and p a factor of a_0 . Procedure *RATFACT* outputs the nonzero rational roots p/q by execution of the procedure whose formal name is *print*. The output procedure uses the string whose formal name is *format* for control of the output format;

```
procedure ratfact (a, n, print, format);
integer array a[0:n]; integer n; procedure print; string
format;
begin integer i, p, q, r, t, f, g;
p loop: for p := 1 step 1 until abs (a[0]) do
  begin comment if p is not a factor of a [0] or q is not a factor
of a[n] then skip to the end of the loop for advance in the
respective for list;
if a[0]  $\neq$  (a[0] $\div$ p) $\times$ p then go to 1
else q loop: for q := 1 step 1 until abs (a[n]) do
  begin if a[n]  $\neq$  (a[n] $\div$ q) $\times$ q then go to 2
  else
  begin comment root test and print;
  comment start polynomial evaluation;
  f := g := a[0]; t := p;
  for i := 1 step 1 until n do
  begin r := a[i] $\times$ t;
  f := f $\times$ q+r;
  g := -g $\times$ q+r;
  t := t $\times$ p;
  end polynomial evaluation;
  comment computing r saves one subscript
evaluation;
if f=0 then print (format, p, q);
if g=0 then print (format,-p, q);
  comment print is the formal name of the procedure
to be used to output the variables in the format
specified by the string whose formal name is format;
  end root test and print;
  2: end q loop;
1: end p loop;
end ratfact, without overflow test.
```

REMARK ON ALGORITHM 78
RATIONAL ROOTS OF POLYNOMIALS WITH
INTEGER COEFFICIENTS [C. Perry, *Comm. ACM*,
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The algorithm was successfully run using the Elliott ALGOL translator on the National-Elliott 803. It was noticed that a multiple rational root will only be printed once by the procedure.

CERTIFICATION OF ALGORITHM 78.
RATFACT (C. Perry, *Comm. ACM* 5, Feb. 1962)
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RATFACT was copied in the Navy Electronics Laboratory International ALGOL Compiler, NELIAC, and tested on the UNIVAC M-490 Countess and the CDC 1604. Polynomials of order 2 through 6 were tested. No corrections were found necessary. It was noted that a polynomial whose coefficients included a common factor would produce superfluous values of p/q , in which this fraction was indeed a root, but one in which p and q contained a common factor.