ALGORITHM 88
EVALUATION OF ASYMPTOTIC EXPRESSION FOR THE FRESNEL SINE AND COSINE INTEGRALS

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real procedure FRESNEL (u) Result: (frcos, frsin); **value** (u);

comment This procedure evaluates the Fresnel sine and cosine integrals for large u by expanding the anymptotic series given by

$$S(u) = \frac{1}{2} - \frac{\cos(x)}{\sqrt{2\pi x}}\left[1 - \frac{1\cdot 3}{(2x)^2} + \frac{1\cdot 3\cdot 5\cdot 7}{(2x)^4} - \cdots\right] - \frac{\sin(x)}{\sqrt{2\pi x}}\left[\frac{1}{2x} - \frac{1\cdot 3\cdot 5}{(2x)^3} + \frac{1\cdot 3\cdot 5\cdot 7\cdot 9}{(2x)^5} - \cdots\right]$$

and

$$C(u) = \frac{1}{2} - \frac{\sin(x)}{\sqrt{2\pi x}}\left[1 - \frac{1\cdot 3}{(2x)^2} + \frac{1\cdot 3\cdot 5\cdot 7}{(2x)^4} - \cdots\right] - \frac{\cos(x)}{\sqrt{2\pi x}}\left[\frac{1}{2x} - \frac{1\cdot 3\cdot 5}{(2x)^3} + \frac{1\cdot 3\cdot 5\cdot 7\cdot 9}{(2x)^5} - \cdots\right]$$

in which $x = \pi u^2/2$. Reference: PEARCEY, T. *Table of the Fresnel Integral to Six Decimal Places.* The Syndics of the Cambridge University Press, Melbourne, Australia (1956).;

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begin   pi := 3.14159265; arg := pi × (u↑2)/2; temp := 1;
        argsq := 1/(4 × (arg↑2)); term := −3 × argsq;
        series := 1 + term; N := 3;
first:  if temp = series then go to second; temp := series;
        termi := term;
        term := −termi × (4 × N − 7) × (4×N − 5) × (argsq);
        if abs(term) > abs(termi) then go to second;
        series := temp + term; N := N + 1; go to first;
second: series2 := ½ × arg; temp := 0; term := series2;
        N := 2;
loop:   if series2 = temp then go to exit; termi := term;
        term := −termi × argsq × (4×N−5) × (4×N−3);
        if abs(term) > abs(termi) then go to exit;
        temp := series2; series2 := temp + term;
        N := N + 1; go to loop;
exit:   if u < 0 then half := −½ else half := ½;
        frcos := half + (sin(arg) × series − cos(arg) + series2)/
        (pi × u);
        frsin := half − (cos(arg) × series2 + sin(arg) × series)/
        (pi × u)
end FRESNEL;
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REMARK ON ALGORITHMS 88, 89 AND 90
EVALUATION OF THE FRESNEL INTEGRALS

[J. L. Cundiff, *Comm. ACM*, May 1962]

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While coding these algorithms in FORTRAN for the IBM 7094, modifications were required (both in the formulation and in the language) before execution with any degree of speed and accuracy could be obtained. In the process it was found that the reference, *Pearcy*, contains an error in the formula for $C(u)$. This error is contained in Algorithm 88 in the formula

$$C(u) = \frac{1}{2} - \frac{\sin(x)}{\sqrt{2\pi x}}[\quad] - \cdots.$$

The first minus sign above should be a plus sign.

After the necessary modifications were made, the three algorithms were found to be too large and uneconomical for our usage. A single algorithm, incorporating these three procedures, was written and is in current usage in a computer program which requires several thousand evaluations of each Fresnel integral.